Hong Kong Mathematics Olympiad (2015/2016)

Final Event 1 (Individual)

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
				+ Bonus score			Time		
				Total score				Min.	Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特别声明，答案须用数字表达，并化至最简。

1. 解方程 $\log_5 a + \log_3 a = \log_5 a \cdot \log_3 a$，其中 $a > 1$ 为实数。

Solve the equation $\log_5 a + \log_3 a = \log_5 a \cdot \log_3 a$ for real number $a > 1$.

$a =$

2. 若 $\sqrt{b} = \sqrt{8+\sqrt{\mathrm{a}}} + \sqrt{8-\sqrt{a}}$，求 b 的实数値。

If $\sqrt{b} = \sqrt{8+\sqrt{\mathrm{a}}} + \sqrt{8-\sqrt{a}}$, determine the real value of b.

$b =$

3. 若方程 $x^2 - cx + b = 0$ 有两实数根及两根之差为 1，求两根之和的最大可能值 c。

If the equation $x^2 - cx + b = 0$ has two distinct real roots and their difference is 1, determine the greatest possible value of the sum of the roots, c.

c =

4. 设 $d = \overline{xyz}$ 为一**不能**被 10 整除的三位数。若 $\overline{xyz}$ 与 $\overline{zyx}$ 之和可被 c 整除，求此整数的最大可能值 d。

Let $d = \overline{xyz}$ be a three-digit integer that is **not** divisible by 10. If the sum of the integers $\overline{xyz}$ and $\overline{zyx}$ is divisible by c, determine the greatest possible value of such an integer d.

d =

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Final Event 2 (Individual)

FOR OFFICIAL USE

Score for accuracy		$\times$	Mult. factor for speed		$=$		Team No.		
				+ Bonus score			Time		
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1. 设一个等边三角形及一个正六边形的周界比率为 $1:1$。若三角形与六边形的面积比率为 $2:a$，求 a 的值。

Let the ratio of the perimeter of an equilateral triangle to the perimeter of a regular hexagon be $1:1$. If the ratio of the area of the triangle to the area of the hexagon is $2:a$, determine the value of a.

a =

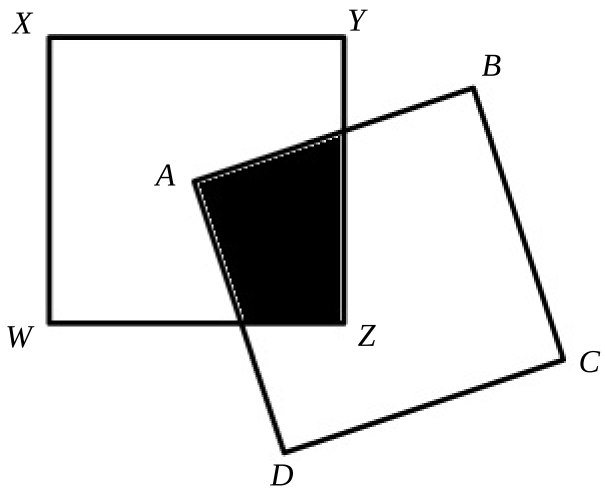
2. 求 $b=\left(\log_2(a^2)+\log_4\left(\frac{1}{a^2}\right)\right)\times\left(\log_a 2+\log_{a^2}\left(\frac{1}{2}\right)\right)$ 的值。

Determine the value of $b=\left(\log_2(a^2)+\log_4\left(\frac{1}{a^2}\right)\right)\times\left(\log_a 2+\log_{a^2}\left(\frac{1}{2}\right)\right)$.

b =

3. 在下图中，正方形 $ABCD$ 及 $XYZW$ 相等而且互相交迭使得顶点 A 位在 $XYZW$ 的中心及线段 AB 将线段 YZ 边分为$1:2$。若 $XYZW$ 的面积与交迭部分的面积比率为 $c:1$，求 c 的值。

In the figure below, identical squares $ABCD$ and $XYZW$ overlap each other in such a way that the vertex A is at the centre of $XYZW$ and the line segment AB cuts line segment YZ into $1:2$. If the ratio of the area of $XYZW$ to the area of the overlapped region is $c:1$, determine the value of c.



c =

4. 若 76 与 d 的最小公倍数 (L.C.M.) 为 456 及 76 与 d 的最大公因子 (H.C.F.) 为 c 求正整数 d 的值。

If the least common multiple (L.C.M.) of 76 and d is 456 and the highest common factor (H.C.F.) of 76 and d is c, determine the value of the positive integer d.

d =

Hong Kong Mathematics Olympiad (2015/2016)
Final Event 3 (Individual)

FOR OFFICIAL USE

Score for accuracy		$\times$	Mult. factor for speed		$=$		Team No.		
			$+$	Bonus score			Time		
			Total score					Min.	Sec.

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1. 若 $f(x) = x^4 + x^3 + x^2 + x + 1$, 求 $f(x^5)$ 除以 $f(x)$ 的余值 a。

If $f(x) = x^4 + x^3 + x^2 + x + 1$, determine the reminder a of $f(x^5)$ divided by $f(x)$.

a =

2. 设 n 为整数。求 $n^a - n$ 除以 30 的余值 b。

Let n be an integer. Determine the remainder b of $n^a - n$ divided by 30.

b =

3.　若 $0 < x < 1$，求

$$c = \left(\frac{\sqrt{1+x}}{\sqrt{1+x} - \sqrt{1-x}} + \frac{1-x}{\sqrt{1-x^2} + x - 1}\right) \times \left(\sqrt{\frac{1}{x^2-b^2} - 1} - \frac{1}{x-b}\right)$$

的值。

If $0 < x < 1$, determine the value of

$$c = \left(\frac{\sqrt{1+x}}{\sqrt{1+x} - \sqrt{1-x}} + \frac{1-x}{\sqrt{1-x^2} + x - 1}\right) \times \left(\sqrt{\frac{1}{x^2-b^2} - 1} - \frac{1}{x-b}\right).$$

c =

4.　若实数 x 及 y 满足方程 $2\log_{10}(x+2cy) = \log_{10} x + \log_{10} y$，

求 $d = \frac{x}{y}$ 的值。

If real numbers x and y satisfy the equation $2\log_{10}(x+2cy) = \log_{10} x + \log_{10} y$,

determine the value of $d = \frac{x}{y}$.

d =

Hong Kong Mathematics Olympiad (2015/2016)

Final Event 4 (Individual)

FOR OFFICIAL USE

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			+	Bonus score			Time		
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除非特别声明，答案须用数字表达，并化至最简。

1. 若 m 和 n 为正整数及 $a=\log_2\left(\left(\frac{m^4n^{-4}}{m^{-1}n}\right)^{-3}\div\left(\frac{m^{-2}n^2}{mn^{-1}}\right)^5\right)$，求 a 的值。

If m and n are positive integers and $a=\log_2\left(\left(\frac{m^4n^{-4}}{m^{-1}n}\right)^{-3}\div\left(\frac{m^{-2}n^2}{mn^{-1}}\right)^5\right)$, determine the value of a.

a =

2. 当整数 1108 + a, 1453, 1844 + 2a 及 2281 除以正整数 n 都得相同余数 b，求 b 的值。

When the integers 1108 + a, 1453, 1844 + 2a and 2281 divided by some positive integer n, they all get the same remainder b. Determine the value of the remainder b.

b =

3. 若 $\frac{6}{b} < x < \frac{10}{b}$, 求 $c = \sqrt{x^2 - 2x + 1} + \sqrt{x^2 - 6x + 9}$ 的最大可能值。

If $\frac{6}{b} < x < \frac{10}{b}$, determine the greatest possible value of $c = \sqrt{x^2 - 2x + 1} + \sqrt{x^2 - 6x + 9}$.

c =

4. 求 $1 + 3^c + (3^c)^2 + (3^c)^3 + (3^c)^4$ 除以 $1 + 3 + 3^2 + 3^3 + 3^4$ 的余值 d。

Determine the remainder d of $1 + 3^c + (3^c)^2 + (3^c)^3 + (3^c)^4$ divided by $1 + 3 + 3^2 + 3^3 + 3^4$.

d =